A Story of Ratios®

Eureka Math™

Grade 8 Module 1

Student File_B

Student Workbook

This file contains:

- G8-M1 Sprint and Fluency Resources[1]
- G8-M1 Exit Tickets
- G8-M1 Mid-Module Assessment
- G8-M1 End-of-Module Assessment

[1]Note that not all lessons in this module include sprint or fluency resources.

Published by the non-profit GREAT MINDS®.

Printed in the U.S.A.
This book may be purchased from the publisher at eureka-math.org
10 9 8 7 6 5 4 3 2

G8-M1-SFB-1.3.1-05.2015

Sprint and Fluency Packet

Number Correct: ______

Applying Properties of Exponents to Generate Equivalent Expressions—Round 1

Directions: Simplify each expression using the laws of exponents. Use the least number of bases possible and only positive exponents. All letters denote numbers.

1.	$2^2 \cdot 2^3$		23.	$6^3 \cdot 6^2$	
2.	$2^2 \cdot 2^4$		24.	$6^2 \cdot 6^3$	
3.	$2^2 \cdot 2^5$		25.	$(-8)^3 \cdot (-8)^7$	
4.	$3^7 \cdot 3^1$		26.	$(-8)^7 \cdot (-8)^3$	
5.	$3^8 \cdot 3^1$		27.	$(0.2)^3 \cdot (0.2)^7$	
6.	$3^9 \cdot 3^1$		28.	$(0.2)^7 \cdot (0.2)^3$	
7.	$7^6 \cdot 7^2$		29.	$(-2)^{12} \cdot (-2)^1$	
8.	$7^6 \cdot 7^3$		30.	$(-2.7)^{12} \cdot (-2.7)^1$	
9.	$7^6 \cdot 7^4$		31.	$1.1^6 \cdot 1.1^9$	
10.	$11^{15} \cdot 11$		32.	$57^6 \cdot 57^9$	
11.	$11^{16} \cdot 11$		33.	$x^6 \cdot x^9$	
12.	$2^{12} \cdot 2^2$		34.	$2^7 \cdot 4$	
13.	$2^{12} \cdot 2^4$		35.	$2^7 \cdot 4^2$	
14.	$2^{12} \cdot 2^6$		36.	$2^7 \cdot 16$	
15.	$99^5 \cdot 99^2$		37.	$16 \cdot 4^3$	
16.	$99^6 \cdot 99^3$		38.	$3^2 \cdot 9$	
17.	$99^7 \cdot 99^4$		39.	$3^2 \cdot 27$	
18.	$5^8 \cdot 5^2$		40.	$3^2 \cdot 81$	
19.	$6^8 \cdot 6^2$		41.	$5^4 \cdot 25$	
20.	$7^8 \cdot 7^2$		42.	$5^4 \cdot 125$	
21.	$r^8 \cdot r^2$		43.	$8 \cdot 2^9$	
22.	$s^8 \cdot s^2$		44.	$16 \cdot 2^9$	

Number Correct: ______

Improvement: ______

Applying Properties of Exponents to Generate Equivalent Expressions—Round 2

Directions: Simplify each expression using the laws of exponents. Use the least number of bases possible and only positive exponents. All letters denote numbers.

1.	$5^2 \cdot 5^3$	
2.	$5^2 \cdot 5^4$	
3.	$5^2 \cdot 5^5$	
4.	$2^7 \cdot 2^1$	
5.	$2^8 \cdot 2^1$	
6.	$2^9 \cdot 2^1$	
7.	$3^6 \cdot 3^2$	
8.	$3^6 \cdot 3^3$	
9.	$3^6 \cdot 3^4$	
10.	$7^{15} \cdot 7$	
11.	$7^{16} \cdot 7$	
12.	$11^{12} \cdot 11^2$	
13.	$11^{12} \cdot 11^4$	
14.	$11^{12} \cdot 11^6$	
15.	$23^5 \cdot 23^2$	
16.	$23^6 \cdot 23^3$	
17.	$23^7 \cdot 23^4$	
18.	$13^7 \cdot 13^3$	
19.	$15^7 \cdot 15^3$	
20.	$17^7 \cdot 17^3$	
21.	$x^7 \cdot x^3$	
22.	$y^7 \cdot y^3$	

23.	$7^3 \cdot 7^2$	
24.	$7^2 \cdot 7^3$	
25.	$(-4)^3 \cdot (-4)^{11}$	
26.	$(-4)^{11} \cdot (-4)^3$	
27.	$(0.2)^3 \cdot (0.2)^{11}$	
28.	$(0.2)^{11} \cdot (0.2)^3$	
29.	$(-2)^9 \cdot (-2)^5$	
30.	$(-2.7)^5 \cdot (-2.7)^9$	
31.	$3.1^6 \cdot 3.1^6$	
32.	$57^6 \cdot 57^6$	
33.	$z^6 \cdot z^6$	
34.	$4 \cdot 2^9$	
35.	$4^2 \cdot 2^9$	
36.	$16 \cdot 2^9$	
37.	$16 \cdot 4^3$	
38.	$9 \cdot 3^5$	
39.	$3^5 \cdot 9$	
40.	$3^5 \cdot 27$	
41.	$5^7 \cdot 25$	
42.	$5^7 \cdot 125$	
43.	$2^{11} \cdot 4$	
44.	$2^{11} \cdot 16$	

Number Correct: ______

Applying Properties of Exponents to Generate Equivalent Expressions—Round 1

Directions: Simplify each expression using the laws of exponents. Use the least number of bases possible and only positive exponents. When appropriate, express answers without parentheses or as equal to 1. All letters denote numbers.

1.	$4^5 \cdot 4^{-4}$		23.	$\left(\frac{1}{2}\right)^6$	
2.	$4^5 \cdot 4^{-3}$		24.	$(3x)^5$	
3.	$4^5 \cdot 4^{-2}$		25.	$(3x)^7$	
4.	$7^{-4} \cdot 7^{11}$		26.	$(3x)^9$	
5.	$7^{-4} \cdot 7^{10}$		27.	$(8^{-2})^3$	
6.	$7^{-4} \cdot 7^{9}$		28.	$(8^{-3})^3$	
7.	$9^{-4} \cdot 9^{-3}$		29.	$(8^{-4})^3$	
8.	$9^{-4} \cdot 9^{-2}$		30.	$(22^0)^{50}$	
9.	$9^{-4} \cdot 9^{-1}$		31.	$(22^0)^{55}$	
10.	$9^{-4} \cdot 9^{0}$		32.	$(22^0)^{60}$	
11.	$5^0 \cdot 5^1$		33.	$\left(\frac{1}{11}\right)^{-5}$	
12.	$5^0 \cdot 5^2$		34.	$\left(\frac{1}{11}\right)^{-6}$	
13.	$5^0 \cdot 5^3$		35.	$\left(\frac{1}{11}\right)^{-7}$	
14.	$(12^3)^9$		36.	$\frac{56^{-23}}{56^{-34}}$	
15.	$(12^3)^{10}$		37.	$\frac{87^{-12}}{87^{-34}}$	
16.	$(12^3)^{11}$		38.	$\frac{23^{-15}}{23^{-17}}$	
17.	$(7^{-3})^{-8}$		39.	$(-2)^{-12} \cdot (-2)^1$	
18.	$(7^{-3})^{-9}$		40.	$\frac{2y}{y^3}$	
19.	$(7^{-3})^{-10}$		41.	$\frac{5xy^7}{15x^7y}$	
20.	$\left(\frac{1}{2}\right)^9$		42.	$\frac{16x^6y^9}{8x^{-5}y^{-11}}$	
21.	$\left(\frac{1}{2}\right)^8$		43.	$(2^3 \cdot 4)^{-5}$	
22.	$\left(\frac{1}{2}\right)^7$		44.	$(9^{-8})(27^{-2})$	

Number Correct: ______

Improvement: ______

Applying Properties of Exponents to Generate Equivalent Expressions—Round 2

Directions: Simplify each expression using the laws of exponents. Use the least number of bases possible and only positive exponents. When appropriate, express answers without parentheses or as equal to 1. All letters denote numbers.

1.	$11^5 \cdot 11^{-4}$		23.	$\left(\frac{3}{7}\right)^5$	
2.	$11^5 \cdot 11^{-3}$		24.	$(18xy)^5$	
3.	$11^5 \cdot 11^{-2}$		25.	$(18xy)^7$	
4.	$7^{-7} \cdot 7^9$		26.	$(18xy)^9$	
5.	$7^{-8} \cdot 7^9$		27.	$(5.2^{-2})^3$	
6.	$7^{-9} \cdot 7^9$		28.	$(5.2^{-3})^3$	
7.	$(-6)^{-4} \cdot (-6)^{-3}$		29.	$(5.2^{-4})^3$	
8.	$(-6)^{-4} \cdot (-6)^{-2}$		30.	$(22^6)^0$	
9.	$(-6)^{-4} \cdot (-6)^{-1}$		31.	$(22^{12})^0$	
10.	$(-6)^{-4} \cdot (-6)^0$		32.	$(22^{18})^0$	
11.	$x^0 \cdot x^1$		33.	$\left(\frac{4}{5}\right)^{-5}$	
12.	$x^0 \cdot x^2$		34.	$\left(\frac{4}{5}\right)^{-6}$	
13.	$x^0 \cdot x^3$		35.	$\left(\frac{4}{5}\right)^{-7}$	
14.	$(12^5)^9$		36.	$\left(\frac{6^{-2}}{7^5}\right)^{-11}$	
15.	$(12^6)^9$		37.	$\left(\frac{6^{-2}}{7^5}\right)^{-12}$	
16.	$(12^7)^9$		38.	$\left(\frac{6^{-2}}{7^5}\right)^{-13}$	
17.	$(7^{-3})^{-4}$		39.	$\left(\frac{6^{-2}}{7^5}\right)^{-15}$	
18.	$(7^{-4})^{-4}$		40.	$\frac{42ab^{10}}{14a^{-9}b}$	
19.	$(7^{-5})^{-4}$		41.	$\frac{5xy^7}{25x^7y}$	
20.	$\left(\frac{3}{7}\right)^8$		42.	$\frac{22a^{15}b^{32}}{121ab^{-5}}$	
21.	$\left(\frac{3}{7}\right)^7$		43.	$(7^{-8} \cdot 49)^{-5}$	
22.	$\left(\frac{3}{7}\right)^6$		44.	$(36^9)(216^{-2})$	

Exit Ticket Packet

Name ____ Date____________________

Lesson 1: Exponential Notation

Exit Ticket

1.

a. Express the following in exponential notation:

$$\underbrace{(-13) \times \cdots \times (-13)}_{35 \text{ times}}.$$

b. Will the product be positive or negative? Explain.

2. Fill in the blank:

$$\underbrace{\frac{2}{3} \times \cdots \times \frac{2}{3}}_{\_\_\_\_\_\_ \text{times}} = \left(\frac{2}{3}\right)^4$$

3. Arnie wrote:

$$\underbrace{(-3.1) \times \cdots \times (-3.1)}_{4 \text{ times}} = -3.1^4$$

Is Arnie correct in his notation? Why or why not?

Name ____ Date____________________

Lesson 2: Multiplication of Numbers in Exponential Form

Exit Ticket

Write each expression using the fewest number of bases possible.

1. Let a and b be positive integers. $23^a \times 23^b =$

2. $5^3 \times 25 =$

3. Let x and y be positive integers and $x > y$. $\frac{11^x}{11^y} =$

4. $\frac{2^{13}}{2^3} =$

Name ____ Date____________________

Lesson 3: Numbers in Exponential Form Raised to a Power

Exit Ticket

Write each expression as a base raised to a power or as the product of bases raised to powers that is equivalent to the given expression.

1. $(9^3)^6 =$

2. $(113^2 \times 37 \times 51^4)^3 =$

3. Let x, y, z be numbers. $(x^2yz^4)^3 =$

4. Let x, y, z be numbers and let m, n, p, q be positive integers. $(x^my^nz^p)^q =$

5. $\frac{4^8}{5^8} =$

Name ____ Date____________________

Lesson 4: Numbers Raised to the Zeroth Power

Exit Ticket

1. Simplify the following expression as much as possible.

$$\frac{4^{10}}{4^{10}} \cdot 7^0 =$$

2. Let a and b be two numbers. Use the distributive law and then the definition of zeroth power to show that the numbers $(a^0 + b^0)a^0$ and $(a^0 + b^0)b^0$ are equal.

Name ____ Date____________________

Lesson 5: Negative Exponents and the Laws of Exponents

Exit Ticket

Write each expression in a simpler form that is equivalent to the given expression.

1. $76543^{-4} =$

2. Let f be a nonzero number. $f^{-4} =$

3. $671 \times 28796^{-1} =$

4. Let a, b be numbers $(b \neq 0)$. $ab^{-1} =$

5. Let g be a nonzero number. $\frac{1}{g^{-1}} =$

Name ____ Date____________________

Lesson 6: Proofs of Laws of Exponents

Exit Ticket

1. Show directly that for any nonzero integer x, $x^{-5} \cdot x^{-7} = x^{-12}$.

2. Show directly that for any nonzero integer x, $(x^{-2})^{-3} = x^{6}$.

Name ____ Date____________________

Lesson 7: Magnitude

Exit Ticket

1. Let $M = 118{,}526.65902$. Find the smallest power of 10 that will exceed M.

2. Scott said that 0.09 was a bigger number than 0.1. Use powers of 10 to show that he is wrong.

Name ____ Date____________________

Lesson 8: Estimating Quantities

Exit Ticket

Most English-speaking countries use the short-scale naming system, in which a trillion is expressed as $1,000,000,000,000$. Some other countries use the long-scale naming system, in which a trillion is expressed as $1,000,000,000,000,000,000,000$. Express each number as a single-digit integer times a power of ten. How many times greater is the long-scale naming system than the short-scale?

Name ____ Date____________________

Lesson 9: Scientific Notation

Exit Ticket

1. The approximate total surface area of Earth is $5.1 \times 10^8 \text{ km}^2$. All the salt water on Earth has an approximate surface area of $352{,}000{,}000 \text{ km}^2$, and all the freshwater on Earth has an approximate surface area of $9 \times 10^6 \text{ km}^2$. How much of Earth's surface is covered by water, including both salt and fresh water? Write your answer in scientific notation.

2. How much of Earth's surface is covered by land? Write your answer in scientific notation.

3. Approximately how many times greater is the amount of Earth's surface that is covered by water compared to the amount of Earth's surface that is covered by land?

Name ____ Date ____________________

Lesson 10: Operations with Numbers in Scientific Notation

Exit Ticket

1. The speed of light is 3×10^8 meters per second. The sun is approximately $230{,}000{,}000{,}000$ meters from Mars. How many seconds does it take for sunlight to reach Mars?

2. If the sun is approximately 1.5×10^{11} meters from Earth, what is the approximate distance from Earth to Mars?

Name ____ Date ________________

Lesson 11: Efficacy of the Scientific Notation

Exit Ticket

1. Two of the largest mammals on earth are the blue whale and the African elephant. An adult male blue whale weighs about 170 tonnes or long tons. (1 tonne $= 1000$ kg)

   Show that the weight of an adult blue whale is 1.7×10^5 kg.

2. An adult male African elephant weighs about 9.07×10^3 kg.

   Compute how many times heavier an adult male blue whale is than an adult male African elephant (i.e., find the value of the ratio). Round your final answer to the nearest one.

Name ______________________________ Date ______________

Lesson 12: Choice of Unit

Exit Ticket

1. The table below shows an approximation of the national debt at the beginning of each decade over the last century. Choose a unit that would make a discussion about the growth of the national debt easier. Name your unit, and explain your choice.

Year	Debt in Dollars
1900	2.1×10^9
1910	2.7×10^9
1920	2.6×10^{10}
1930	1.6×10^{10}
1940	4.3×10^{10}
1950	2.6×10^{11}
1960	2.9×10^{11}
1970	3.7×10^{11}
1980	9.1×10^{11}
1990	3.2×10^{12}
2000	5.7×10^{12}

2. Using the new unit you have defined, rewrite the debt for years 1900, 1930, 1960, and 2000.

Name ____ Date ____________________

Lesson 13: Comparison of Numbers Written in Scientific Notation and Interpreting Scientific Notation Using Technology

Exit Ticket

1. Compare 2.01×10^{15} and 2.8×10^{13}. Which number is larger?

2. The wavelength of the color red is about 6.5×10^{-9} m. The wavelength of the color blue is about 4.75×10^{-9} m. Show that the wavelength of red is longer than the wavelength of blue.

Assessment Packet

Name ______________________________ Date ____________

1. The number of users of social media has increased significantly since the year 2001. In fact, the approximate number of users has tripled each year. It was reported that in 2005 there were 3 million users of social media.

   a. Assuming that the number of users continues to triple each year, for the next three years, determine the number of users in 2006, 2007, and 2008.

   b. Assume the trend in the numbers of users tripling each year was true for all years from 2001 to 2009. Complete the table below using 2005 as year 1 with 3 million as the number of users that year.

Year	-3	-2	-1	0	1	2	3	4	5
# of users in millions					3				

   c. Given only the number of users in 2005 and the assumption that the number of users triples each year, how did you determine the number of users for years 2, 3, 4, and 5?

   d. Given only the number of users in 2005 and the assumption that the number of users triples each year, how did you determine the number of users for years 0, -1, -2, and -3?

e. Write an equation to represent the number of users in millions, N, for year t, $t \geq -3$.

f. Using the context of the problem, explain whether or not the formula $N = 3^t$ would work for finding the number of users in millions in year t, for all $t \leq 0$.

g. Assume the total number of users continues to triple each year after 2009. Determine the number of users in 2012. Given that the world population at the end of 2011 was approximately 7 billion, is this assumption reasonable? Explain your reasoning.

2. Let m be a whole number.

   a. Use the properties of exponents to write an equivalent expression that is a product of unique primes, each raised to an integer power.

$$\frac{6^{21} \cdot 10^{7}}{30^{7}}$$

   b. Use the properties of exponents to prove the following identity:

$$\frac{6^{3m} \cdot 10^{m}}{30^{m}} = 2^{3m} \cdot 3^{2m}$$

   c. What value of m could be substituted into the identity in part (b) to find the answer to part (a)?

3.

a. Jill writes $2^3 \cdot 4^3 = 8^6$ and the teacher marked it wrong. Explain Jill's error.

b. Find n so that the number sentence below is true:

$$2^3 \cdot 4^3 = 2^3 \cdot 2^n = 2^9$$

c. Use the definition of exponential notation to demonstrate why $2^3 \cdot 4^3 = 2^9$ is true.

d. You write $7^5 \cdot 7^{-9} = 7^{-4}$. Keisha challenges you, "Prove it!" Show directly why your answer is correct without referencing the laws of exponents for integers; in other words, $x^a \cdot x^b = x^{a+b}$ for positive numbers x and integers a and b.

Name ______________________________ Date ____________

1. You have been hired by a company to write a report on Internet companies' Wi-Fi ranges. They have requested that all values be reported in feet using scientific notation.

   Ivan's Internet Company boasts that its wireless access points have the greatest range. The company claims that you can access its signal up to $2{,}640$ feet from its device. A competing company, Winnie's Wi-Fi, has devices that extend to up to $2\frac{1}{2}$ miles.

   a. Rewrite the range of each company's wireless access devices in feet using scientific notation, and state which company actually has the greater range ($5{,}280$ feet $= 1$ mile).

   b. You can determine how many times greater the range of one Internet company is than the other by writing their ranges as a ratio. Write and find the value of the ratio that compares the range of Winnie's wireless access devices to the range of Ivan's wireless access devices. Write a complete sentence describing how many times greater Winnie's Wi-Fi range is than Ivan's Wi-Fi range.

c. UC Berkeley uses Wi-Fi over Long Distances (WiLD) to create long-distance, point-to-point links. UC Berkeley claims that connections can be made up to 10 miles away from its device. Write and find the value of the ratio that compares the range of Ivan's wireless access devices to the range of Berkeley's WiLD devices. Write your answer in a complete sentence.

2. There is still controversy about whether or not Pluto should be considered a planet. Although planets are mainly defined by their orbital path (the condition that prevented Pluto from remaining a planet), the issue of size is something to consider. The table below lists the planets, including Pluto, and their approximate diameters in meters.

Planet	*Approximate Diameter* (m)
Mercury	4.88×10^6
Venus	1.21×10^7
Earth	1.28×10^7
Mars	6.79×10^6
Jupiter	1.43×10^8
Saturn	1.2×10^8
Uranus	5.12×10^7
Neptune	4.96×10^7
Pluto	2.3×10^6

a. Name the planets (including Pluto) in order from smallest to largest.

b. Comparing only diameters, about how many times larger is Jupiter than Pluto?

c. Again, comparing only diameters, find out about how many times larger Jupiter is compared to Mercury.

d. Assume you are a voting member of the International Astronomical Union (IAU) and the classification of Pluto is based entirely on the length of the diameter. Would you vote to keep Pluto a planet or reclassify it? Why or why not?

e. Just for fun, Scott wondered how big a planet would be if its diameter was the square of Pluto's diameter. If the diameter of Pluto in terms of meters were squared, what would the diameter of the new planet be? (Write the answer in scientific notation.) Do you think it would meet any size requirement to remain a planet? Would it be larger or smaller than Jupiter?

3. Your friend Pat bought a fish tank that has a volume of 175 liters. The brochure for Pat's tank lists a "fun fact" that it would take 7.43×10^{18} tanks of that size to fill all the oceans in the world. Pat thinks the both of you can quickly calculate the volume of all the oceans in the world using the fun fact and the size of her tank.

a. Given that 1 liter $= 1.0 \times 10^{-12}$ cubic kilometers, rewrite the size of the tank in cubic kilometers using scientific notation.

b. Determine the volume of all the oceans in the world in cubic kilometers using the "fun fact."

c. You liked Pat's fish so much you bought a fish tank of your own that holds an additional 75 liters. Pat asked you to figure out a different "fun fact" for your fish tank. Pat wants to know how many tanks of this new size would be needed to fill the Atlantic Ocean. The Atlantic Ocean has a volume of $323{,}600{,}000$ cubic kilometers.